# Deformation of Magnetic Shells during Magnetic Storms

by

S.-I. Akasofu Geophysical Institute, College, Alaska

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### ABSTRACT

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The nature of the distortion of the earth's magnetic field in the radiation belt regions is studied for the combination of two external sources, namely the ring current and the solar plasma flow. The distortion is expressed by diagrams showing the location of two points on the distorted lines of force, namely the radial distance  $r_e$  of a point at which the line of force crosses the equator and its anchoring latitude  $\emptyset$  (or  $L = 1/\cos^2 \emptyset$ ) on the earth's surface.

The  $r_e$  -  $\emptyset$  (or  $r_e$  - L) diagrams thus constructed are of use in studying the deformations of the outer radiation belt. It is suggested that the lowering of the boundary L value of the outer belt, observed by the Injun I satellite during magnetic storms, is due to the growth of the ring current embedded within the outer belt.

### INTRODUCTION

It has been shown that the parameters L = f (B, I) and the field intensity, B, are useful coordinates for mapping the distribution of magnetically trapped particles McIlwain, 1961. The coordinate system is based on the scalar potential function for the earth's field, which is obtained by a spherical harmonic analysis of the field at the earth's surface. It is known, however, that external sources of magnetic fields can markedly distort the earth's field in the radiation belt regions, although the effects of such distortions at the earth's surface are within the uncertainties of the spherical harmonic analysis.

For instance, the earth's field is often enclosed within a cavity bounded by solar plasma, and thus it: is limited in radial extent Cahill and Amazeen, 1963. The effect of this limitation at the earth's surface is a world-wide increase of the horizontal component of the earth's field, and its magnitude is, in general, not more than  $100 \ \gamma$  - only of order 0.3% of the permanent field. The hypothetical stormtime ring current can also distort the earth's field to a significant extent in the radiation belt regions, but the magnitude of the effect at the earth's surface is, in general, of order 1% of the permanent field. As it is generally

believed that the accuracy of the present spherical harmonic analysis is of order ± 1%, there is a definite limitation to the applicability of the L-B coordinate system, although it is still a quite useful coordinate system for eliminating large longitudinal differences at low L values.

It is therefore necessary to examine the nature and the extent of the distortions produced by plausible external sources and to determine the limitations of the present L-B coordinate system.

One of the conventional ways of visualizing distortions of the earth's field is to find the distorted field lines or deformed magnetic shells and to compare them with the calculated ones based on the potential function. It is very difficult to observe such distortions by direct magnetic field measurements. However, since trapped particles move on actual (distorted) magnetic shells, it is possible to learn something of the form of these shells by simultaneous observation of distinctive features of the distribution of particles in space, using two or more satellites in different orbits.

The Injun I, Explorer XII, Explorer XIV, and Injun III satellites have been used for studies of this nature on the distribution of low energy electrons. O'Brien, Van Allen, Iaughlin, and Frank, 1967 Maehlum and O'Brien, 1963 Freeman, Van Allen, and Cahill, 1963 Frank, Van Allen,

whelpley, and Craven, 1963 One of the distinctive features of the distribution of electrons in the outer belt is the rather sharp outer boundary, beyond which there is a greatly reduced intensity. The situation is shown schematically in Figure 1. For example, the outer boundary is observed at points A and B by Injun I and at points C and D by Explorer XII. If the earth's magnetic field is described precisely by the scalar potential function, the outer boundary must coincide with one of the magnetic shells, labeled by a certain value of L, say L = 8.0. The L values at the points, A, B, C, and D must then be the same, namely L = 8.0.

Machlum and O'Brien  $\boxed{1963}$  suggested, however, that in general the boundary L values obtained by the two satellites do not agree. Such a discrepancy seems to become most serious during the main phase of an intense magnetic storm. During the main phase of the October 28, 1961 magnetic storm, the Injun I satellite observed the boundary at L  $\sim$  4, whereas Explorer XII observed it at L  $\sim$  10 Freeman, 1963. This result strongly suggests that the magnetic shells containing such low energy electrons are sometimes greatly distorted and hence that the L parameter, based on the scalar magnetic potential which assumes curl B = 0 everywhere outside of the solid earth, does not serve as a proper label for a magnetic shell in the simple geometric sense. The present paper presents a study

of the nature of the distortions of the geomegnetic field due to two external sources, the solar plasma flow and the terrestrial ring current, and indicates the way in which observations such as those cited above can be reconciled.

### MODELS

If the earth's field were that of a dipole, a magnetic shell labeled by a given L would be generated by rotating the line of force which crosses the equatorial plane at a radial distance L earth radii from the earth's center. On the earth's surface, loci of constant L (which are the intersections of magnetic shells and the earth's surface) are circles of geomagnetic latitude 0. We note that

$$L\cos^2 \emptyset = 1. (1)$$

We introduce now two external sources of distortion, namely the ring current and the flow of solar plasma. For the ring current, we take the model ring current used by Akasofu and Lin 1963. The parameters for the model are as follows:

 $r_B = 3.0 \text{ a (a = the earth's radius)}$   $g_1 = 2.990$   $g_2 = 0.499$   $\alpha = 2.0$ 

The distribution along an equatorial radius of the particles responsible for the ring current is close to that of the protons reported by Davis and Williamson 1962. Several values of the peak flux were used to examine the effect of the ring current at different stages of its development. The ring

current field  $\triangle$  H<sub>R</sub> along an equatorial radius has already been calculated by Akasofu, Cain, and Chapman /1961/. We denote by  $\triangle$  H<sub>S</sub> the ring current field at the equator at the earth's surface.

The main part of the ring current field outside the ring is dipolar with the magnetic moment  $M_R$  [see Akasofu, Cain, and Chapman, 1961, p. 4025]. Thus the combined equatorial intensity beyond the ring is given by

$$(M_E + M_R)/r_e^3 . (2)$$

In (2),  $M_E$  is the dipolar magnetic moment of the earth and  $r_e$  is the geocentric radial distance.

The flow of solar plasma is affected by the combined field as given by (2). Thus taking the radial distance of the boundary in the equatorial plane to be  $r_B$ , the field intensity  $A H_I$  at a point  $r_e$  along the earth-sun line due to the image dipole is given by

$$\Delta H_T = (M_E + M_R)/(2r_R + r_E)^3$$
 (3)

The plus sign refers to the night side and the minus sign to the day side. Therefore, the total field intensity in the equatorial plane, along the earth-sun line, can be given, as a first approximation, by

$$H_{e} (r_{e}) = \frac{M_{E}}{r_{e}^{3}} + \Delta H_{R} (r_{e}) + \Delta H_{I} (r_{e})$$
 (4)

where we neglect the effects of the image dipole on the ring current ( $\Delta$  H<sub>I</sub> near the center line of the belt,  $r_R$  = 3a, is less than 2% of the dipole field intensity for  $r_B \sim 8a$ ). The field intensity at the earth's surface is the combination of the earth's dipole field and small contributions from the ring current and the image dipole, together with their induction effects.

In applying our calculations to the data obtained by a pair of satellites, such as Injun I and Explorer XII, it is important to know the location of two points on each distorted line of force, namely the point in the equator and that at the earth's surface, or more specifically the radial distance  $r_e$  of the point at which the line of force crosses the equation and the anchoring latitude  $\emptyset$  (or the L value given by  $1/\cos^2 \emptyset$ ) at the earth's surface.

For this purpose, we use the magnetic flux conservation theorem, rather than tracing the distorted lines of force from the equatorial plane to the earth's surface. The method is illustrated for a dipole field as follows.

Taking the spherical coordinates  $(r, \theta, \lambda)$ , the flux  $F_e$  passing through the area ABCD around the earth-sun line in the equatorial plane (Fig. 2) is given by

$$F_{e} = \int H dS = \int \int_{a}^{\lambda} F_{B} H r_{e} dr_{e} d\lambda . \qquad (5)$$

This flux must be equal to the flux  $F_s$  coming into the earth's surface ABEF, namely

$$F_{s} = \int HdS' = \int_{0}^{\lambda} \int_{0}^{\beta} H_{r} a^{2} \cos \beta d \beta d \lambda \qquad (6)$$

where  $0 = \pi/2 - \theta$  and H<sub>r</sub> is the radial component of the field. For the earth's dipole field, (5) and (6) give

$$F_{e} = \int_{a}^{\lambda} d\lambda \int_{a}^{r_{B}} \left(\frac{H_{o}a^{3}}{r_{e}^{3}}\right) r_{e}dr_{e}$$
 (5')

and

$$F_{s} = \int_{0}^{\lambda} d\lambda \int_{0}^{\emptyset} 2H_{o}a^{2} \sin \theta \cos \theta d\theta \qquad (6')$$

where  $H_0$  (= 0.32 gauss) denotes the field intensity at the equator on the earth's surface. Equating (5') and (6'), we find simply the equation of the lines of force, namely

$$a = r_{R} \cos^{2} \emptyset . (7)$$

The approach for the distorted field H is to calculate first the total flux  $F_{\rm e}$  through the equatorial plane and then to

find the latitude,  $\emptyset$ , at which the integrated flux  $F_g$  through the earth's surface becomes the same as  $F_e$ . In order to calculate  $F_g$ , we assume that the field distribution on the earth's surface is given by the combination of the earth's dipole field and a small and uniform ring current field  $\Delta$   $H_R$  and the image dipole field  $\Delta$   $H_T$ . We note that our discussion is limited to the noon-midnight meridian.

## DEFORMATION OF MAGNETIC SHELLS

Under the above scheme, the relation between  $r_e$  (the radial distance of the point at which the line of force crosses the equator) and the anchoring latitude  $\emptyset$  (or  $L = 1/\cos^2 \emptyset$ ) is obtained for various  $A H_S$  (the ring current field at the earth's surface). Hereafter, r,  $r_e$ , and  $r_B$  are given in units of the earth's radius a. Note that L is also expressed in units of a.

Figures 3 and 4 show the  $r_e$  - L (or  $r_e$  -  $\emptyset$ ) relation calculated for  $r_B$  = 8.0 and  $r_B$  = 10.0, respectively, in the manner outlined above.

For the earth's dipole field, the  $r_e$  - L relation in the above figures is expressed by a straight line, making an angle of 17/4 with both the  $r_e$  and L axes. Any deviations from the straight line indicate a distortion of the lines of force from the dipole lines,  $a = r_e \cos^2 \theta$ .

Here, we discuss first the implication of Figures 3 and 4 by use of a schematic diagram, Figure 5, and then return to the former diagrams. Suppose a solar plasma advances towards the earth. As Chapman and Ferraro 1931 showed, there appear two neutral points, one in each hemisphere, at the front of the plasma. In the noon meridian, the lines of force to these neutral points separate the lines of force into two

parts, namely those closed on the sunlit side and those closed on the dark side. When the plasma advances further, the neutral points move towards the equatorial plane. Figure 5a shows schematically this situation when the front of the plasma reaches a point of geocentric distance  $\mathbf{r}_{e} = \mathbf{B}$ . The line of force which originally crossed the equatorial plane at  $\mathbf{r}_{e} = \mathbf{A}$  is now "compressed" and lies just inside the front surface. The line of force which originally crossed the equatorial plane at  $\mathbf{r}_{e} = \mathbf{A}$  is now "transferred" to the night side and closed there.

Such a change of any external field induces an electric current within the solid earth; the induced current tends to prevent the penetration of external fields into the earth. It is known, however, that the storm time fields do penetrate appreciably into the earth's crust. It is estimated that such penetration is effectively terminated on a concentric sphere of radius r a little smaller than that of the earth (r = 0.94a 0.96a; Chapman and Bartels, 1940). However, any changes which come from  $\Delta$  H<sub>I</sub> and  $\Delta$  H<sub>R</sub> and which arise from the fact that the earth is not a perfect conductor, are so small that their effects cannot be shown in our present  $r_e = L$  diagrams. Now, because of the reduction of the radial distance from  $r_e = A$  to  $r_e = B$  and because . the anchoring L value ( $L_1$ ) does not change appreciably, the original straight line is

now curved as is shown in Figure 5b. Similarly, the line of force  $r_e = C$  moves to  $r_e = D$ , but again the anchoring point is unchanged  $(I_p)$ .

Suppose now that the plasma stays at  $r_e = B$  (Figure 5a) and that the ring current begins to grow within an area enclosed by the deformed line of force r = D. The growth of the ring current tends to stretch the lines of force outside the ring towards the boundary see Akasofu, Cain, and Chapman, 1961; their Fig.  $\overline{\mathcal{I}}$ . At a certain epoch of the main phase, this line reaches a point just inside the boundary surface r = B. The lines of force which crossed the equatorial plane between  $r_{a} = D$  and  $r_{a} = B$  in Figure 5 are stretched and then transferred to the night side. Note that because the boundary position is fixed while the ring current is growing, there must occur an increase of the solar plasma pressure in order to balance the growth of the ring current pressure: there is some evidence to support this; at about or a little after the maximum epoch of the main phase of the September 30 and October 28 magnetic storms, the magnetospheric boundary on the day side was closer to the earth than its average position Freeman, 1963. Because  $r_{a} = D$  has moved to  $r_{a} = B$  and because the point I, does not change appreciably, the curve in Figure 5b is now flattened as is seen in Figure 5d.

Both Figure 3 and Figure 4 consist of two parts, one for the day side and one for the night side. A large difference between them is due to the fact that the image dipole effect is larger in the day side. It is of incidental interest that O'Brien 1963 finds a large daily variation of the boundary L values of the outer belt. We note, however, that the image dipole approximation would be less accurate on the night side than on the day side. Further, in order to find the family of lines of force which constitute a magnetic shell for trapped particles, it would be necessary to calculate the second (integral) invariant along the lines of force.

From Figures 3 and 4 it is possible to justify the use of the L-B coordinate system between the earth's surface and  $r_e = 2.5$  within the accuracy of 0.1 for a fairly large magnetic storm. Beyond  $r_e = 3.0$ , however, the deviation from the straight line becomes serious even for a medium storm.

Beyond  $r_e$  = 4.0, the deformation is considerable even for a weak ring current. In order to determine the relation between  $\Delta$  H<sub>S</sub> and the observed field decrease on the earth's surface, we take  $\Delta$  H<sub>S</sub> = -40  $\gamma$  to be the base value for the Dst (H) change  $\sqrt{f}$  or details, see Akasofu, Lin, and Van Allen, 19637. Further, the induced currents make the Dst (H) values larger,

and we take Dst (H') = 1.5 x Dst (H) as the actual observed decrease. The three kinds of scale,  $\triangle$  H<sub>S</sub>, Dst (H), and Dst (H'), are given together is an auxiliary diagram under Figure 3a.

The Explorer XII satellite showed that the outer boundary of the outer radiation belt coincides with the magnetospheric boundary on the day side, where the solar plasma acts on the magnetic field Freeman, Van Allen, and Cahill,  $196\overline{37}$ . Figure 3a shows that for Dst (H')  $\simeq$  -150  $\gamma$ , the lines of force that cross the equator just inside the magnetospheric boundary ( $r_e \simeq 8.0$ ) are anchored at the earth's surface at a point on L = 5.0 or  $\emptyset = 63.5^{\circ}$ .

In order to examine this point more clearly, the line of force anchored at L = 4.8 ( $\theta = 62.5^{\circ}$ ) is calculated under the same conditions, and is shown in Figure 6. Note that it is stretched out to  $r_{\rm e} = 7.8$ . Further, the area enclosed by the distorted line and the earth's surface is shaded to show approximately the cross section of the outer belt in the noon meridian. In this particular case, if two satellites, like Injun I and Explorer XII, observe the boundary in terms of L, the difference in the boundary L values is 7.8 - 4.8 = 3.0. We infer that such a reduction corresponds to the low L value observed by Injun I during the October 28, 1961 magnetic

storm Machlum and O'Brien, 19637. Further, we take this agreement to be important evidence for the existence of the storm-time ring current within the outer radiation belt. It seems that the lowering of the boundary L value observed by Injun I is closely related to the equatorward shift of the auroral strip (in which overhead auroras are seen) and to other related geophysical phenomena during the main phase of magnetic storms Akasofu and Chapman, 1962, 1963; Machlum and O'Brien, 19637.

In Figures 3b and 4b, we see that the discrepancy between r, and L becomes larger on the night side. Therefore at least during magnetic storms, it should not simply be assumed that the L value given by the relation  $L = 1/\cos^2 \emptyset$  represents approximately the radial distance of the point where the line of force anchored at latitude Ø crosses the equatorial plane. The center line of the auroral zone approximately agrees with the L = 6.0 line on the earth's surface. However, it is clear from the above conclusion that during magnetic storms the lines of force anchored in the auroral zone do not cross the equator at a radial distance of r ~ 6.0. During a large main phase, re could be larger than 10.0 on the night side. This seems to be particularly important for the exploration of the origin of auroral particles, mainly soft electrons ( < 10 keV). Further studies are, however, necessary to examine the relation in more detail.

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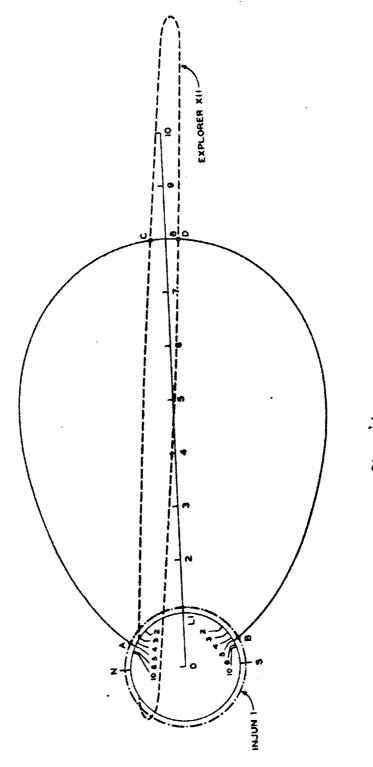
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# FIGURE CAPTIONS

- Figure 1. Schematic diagram to show the orbits of the Injun I and Explorer XII satellites with respect to the outer radiation belt.
- Figure 2. Geometry for the calculation of magnetic flux.
- Figure 3. The  $r_e$  Ø (or  $r_e$  L) relation for various intensities of the ring current  $\triangle H_s$ . The magnetospheric boundary is taken to be  $r_B$  = 8.0. In order to convert  $\triangle H_s$  into actually observed Dst (H'), see the scale below: (a) day side; (b) night side.
- Figure 4. The r<sub>e</sub> Ø (or r<sub>e</sub> L) relation for various intensities of the ring current Δ H<sub>s</sub>. The magnetospheric boundary is taken to be r<sub>B</sub> = 10.0. In order to convert Δ H<sub>s</sub> into actually observed Dot (H'), see the scale given below Figure 3:
  (a) day side; (b) night side.
- Figure 5. Schematic diagram to show the magnetic effects of the solar plasma flow and of the ring current.
- Figure 6. The distorted line of force through the anchoring latitude  $\emptyset = 62.5^{\circ}$  (or L = 4.8), together with the dipole field line crossing the equator at  $r_{\rm e} = 7.8$ . The magnetospheric boundary is at  $r_{\rm B} = 8.0$  and the intensity of the ring current is Dst (H')  $\sim$  -150  $\gamma$ . The area enclosed by the distorted field line is shaded to show approximately the cross section of the outer radiation belt during the main phase of magnetic storms of Dst (H') decrease of order -150  $\gamma$ .



Figure' I

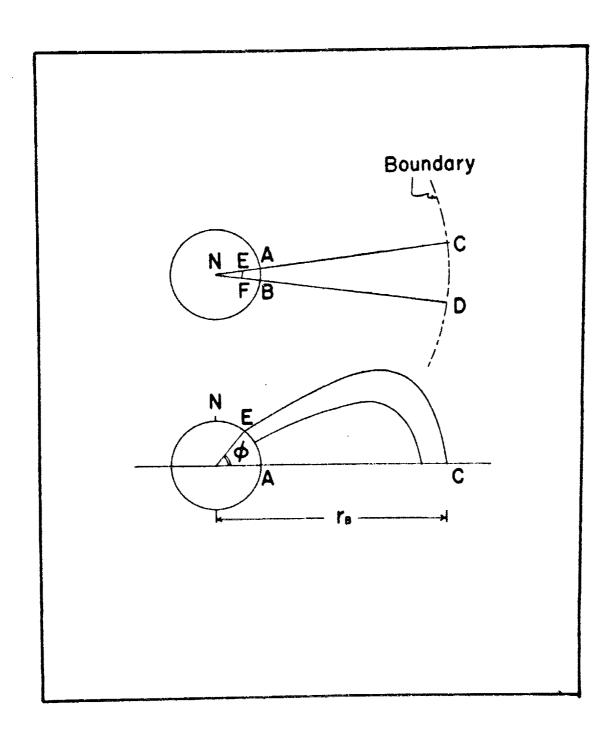
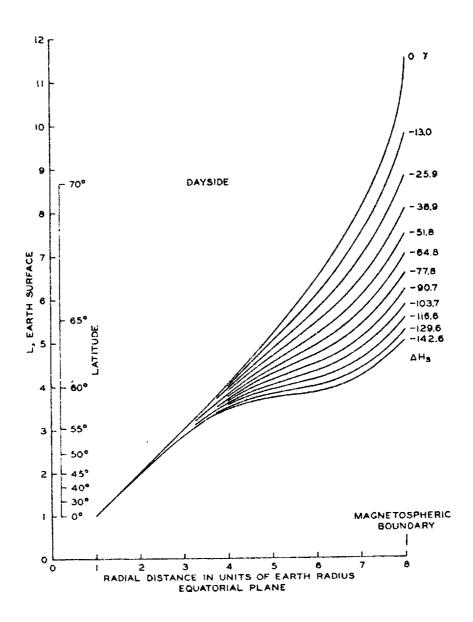


Figure 2



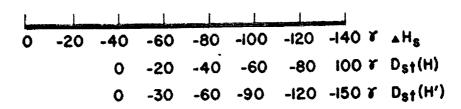


Figure 3a

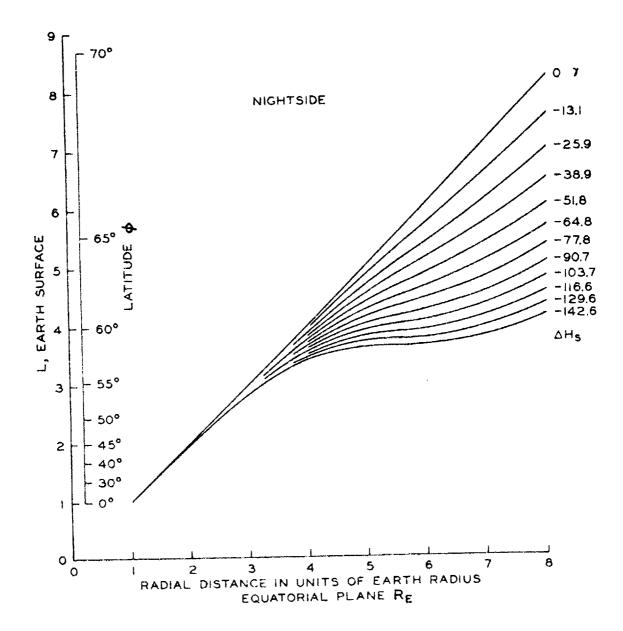


Figure 3b

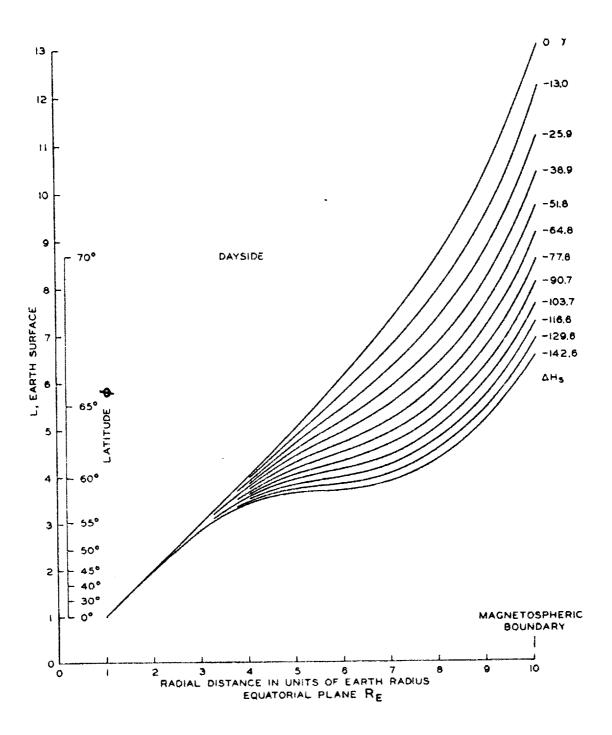


Figure 4a

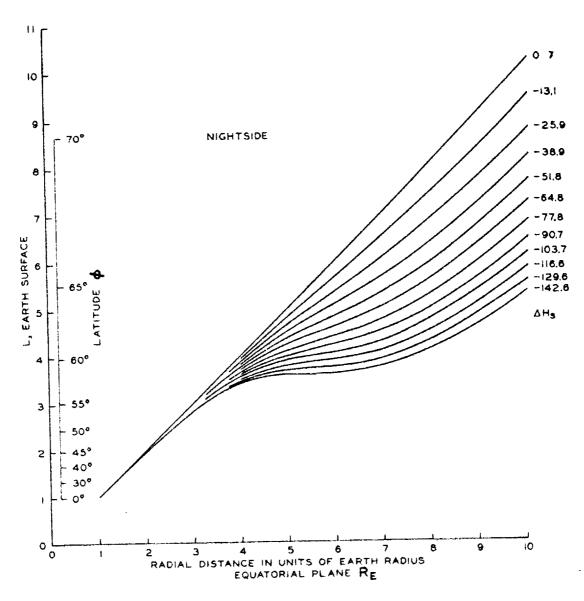


Figure 4b

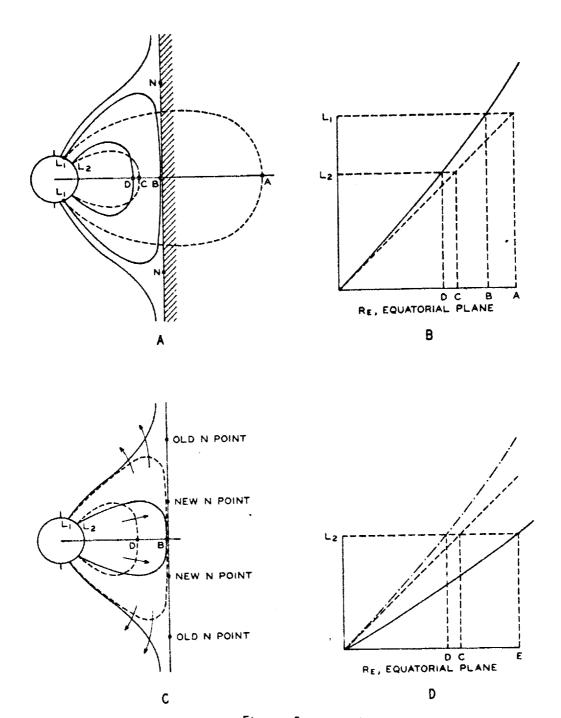


Figure 5

